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Introduction

We are assuming that a small transverse load is placed on a concrete beam with tensile reinforcing and that the load is gradually increased in magnitude until the beam fails. As this takes place, the beam will go through three distinct stages before collapse occurs.

These are:

(1) The un-cracked concrete stage,
(2) The concrete cracked–elastic stresses stage, and
(3) The ultimate-strength stage.
Introduction

Un-cracked Concrete Stage

At small loads when the tensile stresses are less than the *modulus of rupture* (the bending tensile stress at which the concrete begins to crack), the entire cross section of the beam resists bending, with compression on one side and tension on the other. Next figure shows the variation of stresses and strains for these small loads.
Introduction

Concrete Cracked–Elastic Stresses Stage

\[ \varepsilon_c \text{ in compression} \]
\[ f_c \text{ in compression} \]
\[ \varepsilon_s \text{ for steel in tension} \]
\[ \frac{f_s}{n} \text{ (This term is defined in Section 2.3.)} \]

\[ \varepsilon_c \text{ in tension} \]
\[ f_t \text{ tension in concrete} \]

**Figure 2.1** Uncracked concrete stage.
Introduction

Concrete Cracked–Elastic Stresses Stage

As the load is increased beyond the modulus of rupture of the concrete, cracks begin to develop in the bottom of the beam. The moment at which these cracks begin to form—that is, when the tensile stress in the bottom of the beam equals the modulus of rupture— and is referred to as the **cracking moment,** $M_{cr}$. As the load is further increased, these cracks quickly spread up to the vicinity of the neutral axis, and then the neutral axis begins to move upward. The cracks occur at those places along the beam where the actual moment is greater than the cracking moment.
Introduction

Concrete Cracked–Elastic Stresses Stage

Now that the bottom has cracked, another stage is present because the concrete in the cracked zone obviously cannot resist tensile stresses—the steel must do it. This stage will continue as long as the compression stress in the top fibers is less than about one-half of the concrete’s compression strength, $f’c$, and as long as the steel stress is less than its yield stress. The stresses and strains for this range are shown in Figure 2.2(b). In this stage, the compressive stresses vary linearly with the distance from the neutral axis or as a straight line.
FIGURE 2.2  Concrete cracked–elastic stresses stage.
Introduction

Concrete Cracked–Elastic Stresses Stage

The straight-line stress–strain variation normally occurs in reinforced concrete beams under normal service-load conditions because at those loads, the stresses are generally less than 0.50\(f’c\). To compute the concrete and steel stresses in this range, the transformed-area method is used. The service or working loads are the loads that are assumed to actually occur when a structure is in use or service. Under these loads, moments develop that are considerably larger than the cracking moments. Obviously, the tensile side of the beam will be cracked. You will learn to estimate crack widths and methods of limiting their widths.
Introduction

Beam Failure—Ultimate-Strength Stage

As the load is increased further so that the compressive stresses are greater than 0.50$f'$c, the tensile cracks move farther upward, as does the neutral axis, and the concrete compression stresses begin to change appreciably from a straight line. For this initial discussion, it is assumed that the reinforcing bars have yielded. The stress variation is much like that shown in next figure.

To further illustrate the three stages of beam behavior that have just been described, a moment–curvature diagram is shown in next figure.
Introduction

Beam Failure—Ultimate-Strength Stage

FIGURE 2.3  Ultimate-strength stage.
Introduction

Beam Failure—Ultimate-Strength Stage

**FIGURE 2.4** Moment-curvature diagram for reinforced concrete beam with tensile reinforcing only.
Introduction

Beam Failure—Ultimate-Strength Stage

For this diagram, $\Theta$ is defined as the angle change of the beam section over a certain length and is computed by the following expression in which epsilon is the strain in a beam fiber at a distance, $y$, from the neutral axis of the beam:

$$\theta = \frac{\epsilon}{y}$$

The first stage of the diagram is for small moments less than the cracking moment, $M_{cr}$, where the entire beam cross section is available to resist bending. In this range, the strains are small, and the diagram is nearly vertical and very close to a straight line. When the moment is increased beyond the cracking moment, the slope of the curve will decrease a little because the beam is not quite as stiff as it was in the initial stage before the concrete cracked.
Introduction

Beam Failure—Ultimate-Strength Stage
The diagram will follow almost a straight line from $M_{cr}$ to the point where the reinforcing is stressed to its yield point. Until the steel yields, a fairly large additional load is required to appreciably increase the beam’s deflection.

After the steel yields, the beam has very little additional moment capacity, and only a small additional load is required to substantially increase rotations as well as deflections. The slope of the diagram is now very flat.
Cracking Moment
Cracking Moment

The area of reinforcing as a percentage of the total cross-sectional area of a beam is quite small (usually 2% or less), and its effect on the beam properties is almost negligible as long as the beam is un-cracked. Therefore, an approximate calculation of the bending stresses in such a beam can be obtained based on the gross properties of the beam’s cross section. The stress in the concrete at any point a distance $y$ from the neutral axis of the cross section can be determined from the following flexure formula in which $M$ is the bending moment equal to or less than the cracking moment of the section and $I_g$ is the gross moment of inertia of the cross section:

$$f = \frac{My}{I_g}$$
Cracking Moment

Section 9.5.2.3 of the ACI Code states that the cracking moment of a section may be determined with ACI Equation 9-9, in which \( f_r \) is the **modulus of rupture** of the concrete and \( y_t \) is the distance from the centroidal axis of the section to its extreme fiber in tension. In this section, with its equation 9-10, the code states that \( f_r \) may be taken equal to \( 7.5\lambda\sqrt{f'_c} \) with \( f'_c \) in psi.

Or in SI units with \( f'_c \) in N/mm\(^2\) or MPa, \( f_r = 0.7\lambda\sqrt{f'_c} \)

The “lambda” term is 1.0 for normal-weight concrete and is less than 1.0 for lightweight concrete, as described in Section 1.12. The cracking moment is as follows:

\[
M_{cr} = \frac{f_r I_g}{y_t}
\]

(ACI Equation 9-9)
Cracking Moment

The example presents calculations for a reinforced concrete beam where tensile stresses are less than its modulus of rupture. As a result, no tensile cracks are assumed to be present, and the stresses are similar to those occurring in a beam constructed with a homogeneous material.
Cracking Moment

Example 2.1

(a) Assuming the concrete is uncracked, compute the bending stresses in the extreme fibers of the beam of Figure 2.5 for a bending moment of 25 ft-k. The normal-weight concrete has an $f'_c$ of 4000 psi and a modulus of rupture $f_r = 7.5(1.0)\sqrt{4000}$ psi = 474 psi.

(b) Determine the cracking moment of the section.

SOLUTION

(a) Bending stresses:

\[ I_g = \frac{1}{12}bh^3 \text{ with } b = 12 \text{ in. and } h = 18 \text{ in.} \]

\[ I_g = \left(\frac{1}{12}\right)(12 \text{ in.})(18 \text{ in.})^3 = 5832 \text{ in.}^4 \]

\[ f = \frac{My}{I_g} \text{ with } M = 25 \text{ ft-k} = 25,000 \text{ ft-lb} \]

Next, multiply the 25,000 ft-lb by 12 in/ft to obtain in-lb as shown here:

\[ f = \frac{(12 \text{ in/ft} \times 25,000 \text{ ft-lb})(9.00 \text{ in.})}{5832 \text{ in.}^4} = 463 \text{ psi} \]

Since this stress is less than the tensile strength or modulus of rupture of the concrete of 474 psi, the section is assumed not to have cracked.
Cracking Moment

Example 2.1

(b) Cracking moment:

$$M_{cr} = \frac{f_r l_g}{y_t} = \frac{(474 \text{ psi})(5832 \text{ in}^4)}{9.00 \text{ in.}} = 307,152 \text{ in-lb} = 25.6 \text{ ft-k}$$

**Figure 2.5** Beam cross section for Example 2.1.
Cracking Moment

Example 2.2

(a) If the T-beam shown is uncracked, calculate the stress in the concrete at the top and bottom extreme fibers under a positive bending moment of 80 ft-k.

(b) If $f'_c = 3000$ psi and normal-weight concrete is used, what is the maximum uniformly distributed load the beam can carry if it is used as a simple beam with 24-ft span without exceeding the modulus of rupture of the concrete?

(c) Repeat part (b) if the beam is inverted.
Cracking Moment

Example 2.2

(a) Locate the neutral axis with respect to the top of the section:

\[
\bar{y} = \frac{b_f h_f \left( \frac{h_f}{2} \right) + (b_f)(h - h_f) \left( h_f - \frac{h - h_f}{2} \right)}{b_f h_f + (b_f)(h - h_f)}
\]

\[
= \frac{(60 \text{ in.})(5 \text{ in.})(2.5 \text{ in.}) + (12 \text{ in.})(27 \text{ in.}) \left( 5 \text{ in.} + \frac{27 \text{ in.}}{2} \right)}{(60 \text{ in.})(5 \text{ in.}) + (12 \text{ in.})(27 \text{ in.})} = 10.81 \text{ in.}
\]

The moment of inertia is:

\[
l_g = \frac{b_f h_f^3}{12} + b_f h_f \left[ \left( \frac{y - h_f}{2} \right)^2 + \frac{b_w(h - h_f)}{12} + b_w(h - h_f) \right] \left[ \frac{y - h_f - \left( h - h_f \right)}{2} \right]^2
\]

\[
= \frac{(60 \text{ in.})(5 \text{ in.})^3}{12} + (60 \text{ in.})(5 \text{ in.}) \left( 10.81 \text{ in.} - \frac{5 \text{ in.}}{2} \right)^2 + \frac{(12 \text{ in.})(32 \text{ in.} - 5 \text{ in.})^3}{12}
\]

\[
+ (12 \text{ in.})(32 \text{ in.} - 5 \text{ in.}) \left( 10.81 \text{ in.} - 5 \text{ in.} - \frac{27 \text{ in.}}{2} \right)^2
\]

\[
= 60,185 \text{ in.}^4
\]
The stress in the bottom fiber under the given moment of 80 ft-k is:

$$f_{\text{top}} = \frac{Mc}{I} = \frac{(80 \text{ ft-k})(12 \text{ in/ft})(32 \text{ in.} - 10.81 \text{ in.})}{60,185 \text{ in.}^4} = 0.338 \text{ k/in.}^2 = 338 \text{ lb/in.}^2$$

The stress in the top fiber is:

$$f_{\text{top}} = \frac{Mc}{I} = \frac{(80 \text{ ft-k})(12 \text{ in/ft})(10.81 \text{ in.})}{60,185 \text{ in.}^4} = 0.172 \text{ k/in.}^2 = 172 \text{ lb/in.}^2$$

(b) The modulus of rupture, $f_r$, of normal-weight concrete with $f'_c = 3000 \text{ psi}$ is:

$$f_r = 7.5\lambda \sqrt{f'_c} = 7.5(1.0)\sqrt{3000} = 411 \text{ lb/in.}^2$$

The moment that causes a stress equal to the modulus of rupture is:

$$M_{cr} = \frac{f_r l_g}{c} = \frac{(411 \text{ lb/in.}^2)(60,185 \text{ in.}^4)}{(32 \text{ in.} - 10.81 \text{ in.})} = 1167.344 \text{ in-lb} = 97.28 \text{ ft-k}$$

The uniformly distributed load on a simple span that causes this much moment is:

$$w = \frac{8M}{L^2} = \frac{8(97.28 \text{ ft-k})}{(24 \text{ ft})^2} = 1.351 \text{ k/ft} = 1351 \text{ lb/ft}$$
If the beam is inverted, then the $c$ term used to calculate $M_{cr}$ is 10.81 in. instead of 21.19 in., hence:

$$M_{cr} = \frac{f_r l_g}{c} = \frac{(411 \text{ lb/in.}^2)(60,185 \text{ in.}^4)}{(10.81 \text{ in.})} = 2,288,255 \text{ in-lb} = 190.69 \text{ ft-k}$$

The uniformly distributed load on a simple span that causes this much moment is:

$$w = \frac{8M}{L^2} = \frac{8(190.69 \text{ ft-k})}{(24 \text{ ft})^2} = 2.648 \text{ k/ft} = 2648 \text{ lb/ft}$$

This is almost double the load that the beam can carry if oriented the opposite way. Don’t get the impression that this is the best orientation for a T beam, however. In the next section, when we examine reinforced sections, the opposite will be true.
Elastic Stresses—Concrete Cracked
Elastic Stresses—Concrete Cracked

When the bending moment is sufficiently large to cause the tensile stress in the extreme fibers to be greater than the modulus of rupture, it is assumed that all of the concrete on the tensile side of the beam is cracked and must be neglected in the flexure calculations.

The cracking moment of a beam is normally quite small compared to the service load moment. Thus, when the service loads are applied, the bottom of the beam cracks. The cracking of the beam does not necessarily mean that the beam is going to fail. The reinforcing bars on the tensile side begin to pick up the tension caused by the applied moment. On the tensile side of the beam, an assumption of perfect bond is made between the reinforcing bars and the concrete. Thus, the strain in the concrete and in the steel will be equal at equal distances from the neutral axis.
Elastic Stresses—Concrete Cracked

If the strains in the two materials at a particular point are the same, however, their stresses cannot be the same since they have different moduli of elasticity. Thus, their stresses are in proportion to the ratio of their moduli of elasticity. The ratio of the steel modulus to the concrete modulus is called the *modular ratio, n*:

\[ n = \frac{E_s}{E_c} \]

If the modular ratio for a particular beam is 10, the stress in the steel will be 10 times the stress in the concrete at the same distance from the neutral axis. Another way of saying this is that when \( n = 10 \), 1 in.\(^2\) of steel will carry the same total force as 10 in.\(^2\) of concrete.
Elastic Stresses—Concrete Cracked

For the beam in next figure, the steel bars are replaced with an equivalent area of fictitious concrete ($nA_s$), which supposedly can resist tension. This area is referred to as the **transformed area**. The resulting revised cross section or transformed section is handled by the usual methods for elastic homogeneous beams. Also shown in the figure is a diagram showing the stress variation in the beam. On the tensile side, a dashed line is shown because the diagram is discontinuous. There, the concrete is assumed to be cracked and unable to resist tension. The value shown opposite the steel is the fictitious stress in the concrete if it could carry tension. This value is shown as $f_s/n$ because it must be multiplied by $n$ to give the steel stress $f_s$. 
Elastic Stresses—Concrete Cracked

FIGURE 2.6 Cracked, transformed section.
Elastic Stresses—Concrete Cracked

Let us solve few numerical on transformed-area problems to illustrate the calculations necessary for determining the stresses and resisting moments for reinforced concrete beams. The first step to be taken in each of these problems is to locate the neutral axis, which is assumed to be located a distance $x$ from the compression surface of the beam. The first moment of the compression area of the beam cross section about the neutral axis must equal the first moment of the tensile area about the neutral axis. The resulting quadratic equation can be solved by completing the squares or by using the quadratic formula. After the neutral axis is located, the moment of inertia of the transformed section is calculated, and the stresses in the concrete and the steel are computed with the flexure formula.
Elastic Stresses—Concrete Cracked

Example 2.3

Calculate the bending stresses in the beam shown in Figure 2.7 by using the transformed area method, $f_c' = 3000$ psi, $n = 9$, and $M = 70$ ft-k.

SOLUTION

Taking Moments about Neutral Axis (Referring to Figure 2.8)

\[(12 \text{ in.}) (x) \left( \frac{x}{2} \right) = (9) (3.00 \text{ in.})(17 \text{ in.} - x)\]

\[6x^2 = 459 - 27.00x\]

Solving by Completing the Square

\[6x^2 + 27.00x = 459\]

\[x^2 + 4.50x = 76.5\]

\[(x + 2.25)(x + 2.25) = 76.5 + (2.25)^2\]

\[x = 2.25 + \sqrt{76.5 + (2.25)^2}\]

\[x = 6.780 \text{ in.}\]
Elastic Stresses—Concrete Cracked

Example 2.3

Moment of Inertia

\[ I = \left(\frac{1}{3}\right)(12 \text{ in.})(6.78 \text{ in.})^3 + (9)(3.00 \text{ in.}^2)(10.22 \text{ in.})^2 = 4067 \text{ in.}^4 \]

Bending Stresses

\[ f_c = \frac{M_y}{I} = \frac{(12)(70,000 \text{ ft-lb})(6.78 \text{ in.})}{4067 \text{ in.}^4} = 1400 \text{ psi} \]

\[ f_s = n\frac{M_y}{I} = (9)(12)(70,000 \text{ ft-lb})(10.22 \text{ in.})}{4067 \text{ in.}^4} = 18,998 \text{ psi} \]

\[ \text{FIGURE 2.7} \quad \text{Beam cross section for Example 2.3.} \]

\[ \text{FIGURE 2.8} \quad \text{Cracked, transformed section for Example 2.3.} \]
Elastic Stresses—Concrete Cracked

Example 2.4

Determine the allowable resisting moment of the beam of Example 2.3, if the allowable stresses are $f_c = 1350$ psi and $f_s = 20,000$ psi.

SOLUTION

\[
M_c = \frac{f_c l}{y} = \frac{(1350 \text{ psi})(4067 \text{ in.}^4)}{6.78 \text{ in.}} = 809,800 \text{ in-lb} = 67.5 \text{ ft-k}
\]

\[
M_s = \frac{f_s l}{ny} = \frac{(20,000 \text{ psi})(4067 \text{ in.}^4)}{(9)(10.22 \text{ in.})} = 884,323 \text{ in-lb} = 73.7 \text{ ft-k}
\]
Elastic Stresses—Concrete Cracked

Discussion

For a given beam, the concrete and steel will not usually reach their maximum allowable stresses at exactly the same bending moments. Such is the case for this example beam, where the concrete reaches its maximum permissible stress at 67.5 ft-k, while the steel does not reach its maximum value until 73.7 ft-k is applied. The resisting moment of the section is 67.5 ft-k because if that value is exceeded, the concrete becomes overstressed even though the steel stress is less than its allowable stress.
Elastic Stresses—Concrete Cracked

Example 2.5

Compute the bending stresses in the beam shown in Figure 2.9 by using the transformed-area method; \( n = 8 \) and \( M = 110 \text{ ft}-\text{k} \).

**SOLUTION**

**SOLUTION**

Locating Neutral Axis (Assuming Neutral Axis below Hole)

\[
\begin{align*}
(18 \text{ in.})(x)\left(\frac{x}{2}\right) - (6 \text{ in.})(6 \text{ in.})(x - 3 \text{ in.}) &= (8)(5.06 \text{ in.}^2)(23 \text{ in.} - x) \\
9x^2 - 36x + 108 &= 931 - 40.48x \\
9x^2 + 4.48x &= 823 \\
x^2 + 0.50x &= 91.44 \\
(x + 0.25)(x + 0.25) &= 91.44 + (0.25)^2 = 91.50 \\
x + 0.25 &= \sqrt{91.50} = 9.57 \\
x &= 9.32 \text{ in.} > 6 \text{ in.} \quad \therefore \text{N.A. below hole as assumed}
\end{align*}
\]
Elastic Stresses—Concrete Cracked

Example 2.5

**Moment of Inertia**

\[ l = \left( \frac{1}{3} \right) (6 \text{ in.})(9.32 \text{ in.})^3(2) + \left( \frac{1}{3} \right) (6 \text{ in.})(3.32 \text{ in.})^3 + (8)(5.06 \text{ in.}^2)(13.68 \text{ in.})^2 = 10,887 \text{ in.}^4 \]

**Computing Stresses**

\[ f_c = \frac{(12)(110,000 \text{ ft-lb})(9.32 \text{ in.})}{10,887 \text{ in.}^4} = 1130 \text{ psi} \]

\[ f_s = \frac{(8)(12)(110,000 \text{ ft-lb})(13.68 \text{ in.})}{10,887 \text{ in.}^4} = 13,269 \text{ psi} \]

**Figure 2.9** Beam cross section for Example 2.5.
Elastic Stresses—Concrete Cracked

Next example will illustrate the analysis of a doubly reinforced concrete beam—that is, one that has compression steel as well as tensile steel. Compression steel is generally thought to be uneconomical, but occasionally its use is quite advantageous.

Compression steel permits the use of appreciably smaller beams than those that make use of tensile steel only. Reduced sizes can be very important where space or architectural requirements limit the sizes of beams. Compression steel is quite helpful in reducing long-term deflections, and such steel is useful for positioning stirrups or shear reinforcement. A detailed discussion of doubly reinforced beam will follow.
Elastic Stresses—Concrete Cracked

Should the compression side of a beam be reinforced, the long-term stresses in that reinforcing will be greatly affected by the creep in the concrete. As time goes by, the compression concrete will compact more tightly, leaving the reinforcing bars (which themselves have negligible creep) to carry more and more of the load.

As a consequence of this creep in the concrete, the stresses in the compression bars computed by the transformed-area method are assumed to double as time goes by.
To be precise, it will be noted in the example that the compression steel area is really multiplied by $2n - 1$. The transformed area of the compression side equals the gross compression area of the concrete plus $2nA'$ minus the area of the holes in the concrete ($1A'$), which theoretically should not have been included in the concrete part. This equals the compression concrete area plus $(2n - 1)A'$. Similarly, $2n - 1$ is used in the moment of inertia calculations. The stresses in the compression bars are determined by multiplying $2n$ times the stresses in the concrete located at the same distance from the neutral axis.
Elastic Stresses—Concrete Cracked

Example 2.7

Compute the bending stresses in the beam shown in Figure 2.10; \( n = 10 \) and \( M = 118 \) ft-k.

**SOLUTION**

**Locating Neutral Axis**

\[
(14 \text{ in.})(x) \left( \frac{x}{2} \right) + (20 - 1)(2.00 \text{ in.}^2)(x - 2.5 \text{ in.}) = (10)(4.00 \text{ in.}^2)(17.5 \text{ in.} - x)
\]

\[
7x^2 + 38x - 95 = 700 - 40x
\]

\[
7x^2 + 78x = 795
\]

\[
x^2 + 11.14x = 113.57
\]

\[
x + 5.57 = \sqrt{113.57 + (5.57)^2} = 12.02
\]

\[
x = 6.45 \text{ in.}
\]

**Moment of Inertia**

\[
l = \left( \frac{1}{3} \right)(14 \text{ in.})(6.45 \text{ in.})^3 + (20 - 1)(2.00 \text{ in.}^2)(3.95 \text{ in.})^2 + (10)(4.00 \text{ in.}^2)(11.05 \text{ in.})^2
\]

\[
= 6729 \text{ in.}^4
\]
Elastic Stresses—Concrete Cracked

Example 2.7

Bending Stresses

\[ f_c = \frac{(12)(118,000 \text{ ft-lb})(6.45 \text{ in.})}{6729 \text{ in.}^4} = 1357 \text{ psi} \]

\[ f_s = 2n \frac{My}{I} = (2)(10) \frac{(12)(118,000 \text{ ft-lb})(3.95 \text{ in.})}{6729 \text{ in.}^4} = 16,624 \text{ psi} \]

\[ f_s = (10) \frac{(12)(118,000 \text{ ft-lb})(11.05 \text{ in.})}{6729 \text{ in.}^4} = 23,253 \text{ psi} \]

**FIGURE 2.10** Beam cross section for Example 2.7.
Ultimate or Nominal Flexural Moments
Ultimate or Nominal Flexural Moments

A very brief introduction to the calculation of the ultimate or nominal flexural strength will be discussed. This topic will be discussed at length in the next chapter, where formulas, limitations, designs, and other matters are presented. For this discussion, it is assumed that the tensile reinforcing bars are stressed to their yield point before the concrete on the compressive side of the beam is crushed. ACI Code requires all beam designs to fall into this category.

After the concrete compression stresses exceed about 0.50$f'c$, they no longer vary directly as the distance from the neutral axis or as a straight line. Rather, they vary much as shown in next figure.
Ultimate or Nominal Flexural Moments

**FIGURE 2.11** Compression and tension couple at nominal moment.
Ultimate or Nominal Flexural Moments

It is assumed for the purpose of this discussion that the curved compression diagram can be replaced with a rectangular one with a constant stress of $0.85f'c$, as shown in part (c) of the figure. The rectangular diagram of depth $a$ is assumed to have the same c.g. (center of gravity) and total magnitude as the curved diagram. These assumptions will enable us to easily calculate the theoretical or nominal flexural strength of reinforced concrete beams. Experimental tests show that with the assumptions used here, accurate flexural strengths are determined.
Ultimate or Nominal Flexural Moments

To obtain the nominal or theoretical moment strength of a beam, the simple steps to follow are:

1. Compute total tensile force $T = A_s f_y$.

2. Equate total compression force $C = 0.85 f'_c a b$ to $A_s f_y$ and solve for $a$. In this expression, $ab$ is the assumed area stressed in compression at $0.85 f'_c$. The compression force $C$ and the tensile force $T$ must be equal to maintain equilibrium at the section.

3. Calculate the distance between the centers of gravity of $T$ and $C$. (For a rectangular beam cross section, it equals $d - a/2$.)

4. Determine $M_n$, which equals $T$ or $C$ times the distance between their centers of gravity.
Ultimate or Nominal Flexural Moments

Example 2.8

Determine $M_n$, the nominal or theoretical ultimate moment strength of the beam section shown in Figure 2.12, if $f_y = 60,000$ psi and $f'_c = 3000$ psi.

**SOLUTION**

**Computing Tensile and Compressive Forces $T$ and $C$**

$$T = A_s f_y = (3.00 \text{ in.}^2)(60 \text{ k/ln.}^2) = 180 \text{ k}$$

$$C = 0.85 f'_c ab = (0.85)(3 \text{ k/ln.}^2)(a)(14 \text{ in.}) = 35.7a$$

**Equating $T$ and $C$ and Solving for $a$**

$$T = C \text{ for equilibrium}$$

$$180 \text{ k} = 35.7a$$

$$a = 5.04 \text{ in.}$$

**Computing the Internal Moment Arm and Nominal Moment Capacity**

$$d - \frac{a}{2} = 21 \text{ in.} - \frac{5.04 \text{ in.}}{2} = 18.48 \text{ in.}$$

$$M_n = (180 \text{ k})(18.48 \text{ in.}) = 3326.4 \text{ in-k} = \boxed{277.2 \text{ ft-k}}$$
Ultimate or Nominal Flexural Moments

Example 2.8

FIGURE 2.12 Beam cross section for Example 2.8.
Ultimate or Nominal Flexural Moments

In next example, the nominal moment capacity of another beam is determined much as it was in previous example. The only difference is that the cross section of the compression area \((A_c)\) stressed at \(0.85f'_c\) is not rectangular. As a result, once this area is determined, we need to locate its center of gravity. The c.g. for the beam is shown as being a distance \(y\) from the top of the beam. The lever arm from \(C\) to \(T\) is equal to \(d - \bar{y}\) (which corresponds to \(d - a/2\)) and \(M_n\) equals \(A_s f_y (d - \bar{y})\).

With this very simple procedure, values of \(M_n\) can be computed for tensile reinforced beams of any cross section.
Ultimate or Nominal Flexural Moments

Example 2.9

Calculate the nominal or theoretical ultimate moment strength of the beam section shown in Figure 2.13, if $f_y = 60,000$ psi and $f_c' = 3000$ psi. The 6-in.-wide ledges on top are needed for the support of precast concrete slabs.

**SOLUTION**

\[
T = A_s f_y = (4.00 \text{ in.}^2) (60 \text{ k/in.}^2) = 240 \text{ k}
\]
\[
C = (0.85f_c')(\text{area of concrete } A_c \text{ stressed to } 0.85f_c')
\]
\[
= 0.85f_c'A_c
\]

Equating $T$ and $C$ and Solving for $A_c$

\[
A_c = \frac{T}{0.85f_c'} = \frac{240 \text{ k}}{(0.85)(3 \text{ k/in.}^2)} = 94.12 \text{ in.}^2
\]

The top 94.12 in.$^2$ of the beam in Figure 2.14 is stressed in compression to $0.85f_c'$. This area can be shown to extend 9.23 in. down from the top of the beam. Its c.g. is located by taking moments at the top of the beam as follows:

\[
\bar{y} = \frac{(36 \text{ in.}^2)(3 \text{ in.}) + (58.12 \text{ in.}^2)(6 \text{ in.} + \frac{3.23 \text{ in.}}{2})}{94.12} = 5.85 \text{ in.}
\]
\[
d - \bar{y} = 21 \text{ in.} - 5.85 \text{ in.} = 15.15 \text{ in.}
\]
\[
M_n = (240 \text{ k})(15.15 \text{ in.}) = 3636 \text{ in-k} = 303 \text{ ft-k}
\]
Ultimate or Nominal Flexural Moments

**Figure 2.13** Beam cross section for Example 2.9.

**Figure 2.14** Area under compression stress block for Example 2.9.